Computation (40 points)

1. (10 points) Differentiate. You do not need to simplify your answers.

(a) \( f(x) = 4x^5 - 3x^7 + 2 + x^{-5/3} \)

(b) \( F(x) = \ln(x^2 + 1) \)

(c) \( f(x) = \frac{5x + 1}{x^2 - 2} \)

(d) \( f(x) = \frac{1}{x} \)

(e) \( g(x) = x^5(4x^3 - 7x^2 + 9)^4 \)

2. (2 points) Compute \( \sum_{i=3}^{6} i^2 \).
3. (10 points) Integrate. In the case of definite integrals, you must show all steps; a numerical answer from your calculator is not sufficient. (That is, you must convince me you know how to find the definite integrals without your calculator.)

(a) \[ \int (x^2 - 4x^3) \, dx \]

(b) \[ \int_{1}^{25} \frac{1}{\sqrt{x^5}} \, dx \]

(c) \[ \int x^3(x^4 + 4)^8 \, dx \]

(d) \[ \int e^{-7x} \, dx \]

(e) \[ \int_{0}^{5} \frac{2x}{x^2 + 1} \, dx \]

NOTE: For the rest of the exam, you may use your calculator to compute any definite integrals that arise.

4. (2 points) Solve \( 24(0.8)^{t/6} = 12 \) for \( t \).

5. (2 points) Find the average rate of change of \( F(x) = \ln(x^2 + 1) \) on the interval \([0, 5]\).
6. (8 points) Compute each sum, if possible. If it is not possible, explain why not.

(a) \( 3 + 3 \cdot 1.5 + 3 \cdot (1.5)^2 + 3 \cdot (1.5)^3 + \ldots + 3 \cdot (1.5)^{17} \).

(b) \( 2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \ldots \)

(c) \( 1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \ldots \)

(d) \( 1 + 1.2 + (1.2)^2 + (1.2)^4 + \ldots \)

7. (4 points) Let \( f(x) = 2x^3 + 3x^2 - 72x + 20 \). Use techniques of calculus to find the local and absolute extrema of \( f \) on \([-4, 4]\). (I.e., don’t just use your calculator!)

8. (2 points) Find the average value of \( f(x) = \frac{2x}{x^2 + 1} \) on \([0, 5]\).
Concepts (40 points)

9. (7 points) The graph of $y = f(x)$ is shown below.

(a) (2 points) Put a small square around the local maxima of $f$, and put a small circle around the local minima of $f$.

(b) (2 points) Draw a small triangle around the inflection points of $f$.

(c) (1 point) On the graph of $f$, draw a small X through the points at which $f'$ is a maximum or a minimum.

(d) (2 points) Sketch a graph of $y = f'(x)$ on the same set of axes.

![Graph of $y = f(x)$]

10. (3 points) Estimate the rate of change of $f(x) = 5x^2$ at $x = 2$ using $h = 0.01$. Compare this to the actual rate of change of $f$ at $x = 2$.

11. (3 points) Find an equation of the tangent line to the graph of $y = f(x)$, where $f(x) = \ln(x)$, at $x = 1$. Sketch a graph illustrating both $f$ and this tangent line.

12. (3 points) If $F(0) = 2$ and $F'(x) = \frac{x^3}{4}$, what is $F(4)$?
13. (3 points) What is the relationship among Numbers 1b, 3e, 5, and 8?

14. (3 points) Suppose that the average value of $f$ on the interval $[1, 4]$ is 7. What is 
\[ \int_1^4 f(x) \, dx \]  
How do you know?

15. (3 points) Suppose that the average rate of change of $F$ on the interval $[1, 4]$ is 7. What is the net change in $F$ over the interval $[1, 4]$? How do you know?

16. (2 points) On the graph of $y = f(x)$ shown below, illustrate the average value of $f$ over $[0, 3]$.

![Graph of $y = f(x)$](image)

17. (3 points) Use the definition of the derivative to find the derivative of $f(x) = -3x^2$. 

18. (6 points) Define $F(x)$ by $F(x) = \int_0^x f(t)dt$, where $y = f(x)$ is the graph shown.

(a) Illustrate $F(5)$ graphically.

(b) What is the numerical value of $F(5)$?

(c) What is $F'(5)$?

19. (2 points) The graph shown below is the graph of $y = F'(x)$. Given that $F(0) = 0$, for what other values of $b$ is $F(b) = 0$?

20. (2 points) Suppose that the graph of $f$ is concave down at $x = c$. What can you conclude about $f'$ at $x = c$?

Applications (20 points)

21. (5 points) You take out a $10,000 business loan to start your new doorknob polishing company. If the interest rate is 12% annually, compounded monthly, and you make monthly payments of $150, how long will it take to pay off the loan? You must show your work to receive credit!
22. (5 points) The amount of a certain drug in a patient’s bloodstream $t$ hours after taking the drug is given by $A(t) = te^{-t/5}$. At what time is the amount in the bloodstream the greatest?

23. (5 points) A police officer’s radar gun clocks a motorist travelling

$$v(t) = \frac{1}{3600} \left( \frac{10(t^2 + 2)}{t^2 + 1} + 60 \right)$$

miles per second $t$ seconds after the officer begins monitoring. How far does the motorist travel in one minute?

24. (5 points) Suppose that the federal government is giving an $8$ billion tax rebate. Assuming that people spend 75% of the money they receive (from any source), how much additional spending is generated?

Congratulations! I hope to have grades posted by Wednesday afternoon. Have a great break and a wonderful Christmas!