Solutions to Homework Assignment 3

1. (a) \( y = f(x) + 3 \)  
   (d) \( y = f(x) + 3 \)  
   (g) \( y = 3f(x) \)  
(b) \( y = f(x) - 3 \)  
   (e) \( y = -f(x) \)  
   (h) \( y = \frac{1}{3}f(x) \)  
(c) \( y = f(x) - 3 \)  
   (f) \( y = f(-x) \)  

6. We need to shift right 2 units and stretch vertically by a factor of 2: \( y = 2\sqrt{3(x - 2)} - (x - 2)^2 \).

7. We need to reflect across the x-axis, shift down 1 unit, and shift 4 units: \( y = -\sqrt{3(x + 4)} - (x + 4)^2 - 1 \).

22. The tangent graph is shifted right by \( \pi/4 \) and compressed vertically by a factor of 4.

23. All portions of the graph of \( \sin x \) that lie below the x-axis are reflected above the x-axis; other portions are unchanged.

![Graphs of functions](image)

29. \((f + g)(x) = f(x) + g(x) = (x^3 + 2x^2) + (3x^2 - 1) = x^3 + 5x^2 - 1\), \((f - g)(x) = f(x) - g(x) = (x^3 + 2x^2) - (3x^2 - 1) = x^3 - x^2 + 1\), \((fg)(x) = f(x)g(x) = (x^3 + 2x^2)(3x^2 - 1)\), and \(\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^3 + 2x^2}{3x^2 - 1}\).

\( f + g, f - g, \) and \( fg \) all have domain \( \mathbb{R} \). \( \frac{f}{g} \) has domain \( \{x \in \mathbb{R} | x \neq \pm 1/\sqrt{3}\} \).

35. \((f \circ g)(x) = f(g(x)) = f\left(\frac{x + 1}{x + 2}\right) = \left(\frac{x + 1}{x + 2}\right) + \left(\frac{1}{\frac{x + 1}{x + 2}}\right) = \frac{x + 1}{x + 2} + \frac{x + 2}{x + 1}\).

\((g \circ f)(x) = g\left(\frac{1}{x}\right) = \frac{x + 1}{x + \frac{1}{x} + 2} = \frac{x^2 + x + 1}{x^2 + 2x + 1}\).

You can get the other two!

37. \((f \circ g \circ h)(x) = f(g(h(x))) = f(g(x - 1)) = f(2(x - 1)) = 2(x - 1) + 1 = 2x - 1\).

41. The “inside” function is \( g \), so \( g(x) = x^2 + 1 \). The outside function is \( f(x) = x^{10} \). Thus \( F(x) = (f \circ g)(x) = f(g(x)) = f(x^2 + 1) = (x^2 + 1)^{10} \), as desired.

49. The innermost function, \( h \), is \( h(x) = \sqrt{x} \). The intermediate function is \( g(x) = \sec x \). The outermost function is \( f(x) = x^4 \). We have \( H(x) = (f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{x})) = f(\sec(\sqrt{x})) = (\sec(\sqrt{x}))^4 = \sec^4(\sqrt{x}) \), as desired.

51. (a) \( g(2) \) is 5, so \( f(g(2)) = f(5) = 4 \).
   (b) \( f(0) = 0 \), so \( g(f(0)) = g(0) = 3 \).
   (c) \( (f \circ g)(0) = f(g(0)) = f(3) = 0 \).
   (d) \( (g \circ f)(6) = g(f(6)) = g(6) \), which is undefined.
   (e) \( (g \circ g)(-2) = g(g(-2)) = g(1) = 4 \).
(f) \((f \circ f)(4) = f(f(4)) = f(2) = -2\).

56. (a) The horizontal distance is just \(d = 350t\).

(b) The distance \(s\) is the hypotenuse of a right triangle with short leg 1 mile and long leg \(d\) miles. Thus, \(s = \sqrt{d^2 + 1}\).

(c) We have \(s(d(t)) = s(350t) = \sqrt{350^2t^2 + 1}\).