Solutions to Homework Assignment 13

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2-7, 13, 16, 17, 19-26, 28, 29, 33

2. Since the slopes of the tangent lines are all positive, \( f'(2) > 0 \). Since \( f \) is increasing, \( f(3) - f(2) > 0 \) and \( \frac{1}{2}(f(4) - f(2)) > 0 \). Thus, 0 is the smallest. Notice that \( f(3) - f(2) \) is the rise in the function as \( x \) goes from 2 to 3. The tangent line at \( x = 2 \) will lie above the graph since the graph bends downward, so the rise of the tangent line as \( x \) goes from 2 to 3, which is \( f'(2) \), will be greater than the corresponding rise in the function. This means that \( f'(2) > f(3) - f(2) \). Likewise, \( \frac{1}{2}(f(4) - f(2)) \) is the slope of the secant line joining \((2, f(2))\) and \((4, f(4))\), which is even less than \( f(3) - f(2) \). Our order is thus 0, \( \frac{1}{2}(f(4) - f(2)) \), \( f(3) - f(2) \), \( f'(2) \).

4. Since \((4, 3)\) is on the graph, \( f(4) = 3 \). The slope of the tangent line is \( \frac{3 - 2}{4 - 0} = \frac{1}{4} \) since the tangent line passes through \((4, 3)\) and \((0, 2)\). Thus, \( f'(4) = \frac{1}{4} \).

5. We need to arrange the tangent lines to match these slopes; the graph below is one possibility.

13.

\[
f'(a) = \lim_{x \to a} \frac{(3 - 2x + 4x^2) - (3 - 2a + 4a^2)}{x - a}
\]

\[
= \lim_{x \to a} \frac{3 - 2x + 4x^2 - 3 + 2a - 4a^2}{x - a}
\]

\[
= \lim_{x \to a} \frac{2(a - x) + 4(x^2 - a^2)}{x - a}
\]

\[
= \lim_{x \to a} \frac{-2(x - a) + 4(x + a)(x - a)}{x - a}
\]

\[
= \lim_{x \to a} \frac{4(x + a) - 2(x - a)}{x - a}
\]

\[
= \lim_{x \to a} 4(x + a) - 2
\]

\[
= 8a - 2.
\]
17. 
\[
f'(a) = \lim_{x \to a} \frac{1}{\sqrt{x+2}} - \frac{1}{\sqrt{a+2}} \frac{\sqrt{a+2} - \sqrt{x+2}}{x - a} \\
= \lim_{x \to a} \frac{(\sqrt{a+2} - \sqrt{x+2})(\sqrt{a+2} + \sqrt{x+2})}{(a+2)(x+2)(x-a)(\sqrt{a+2} + \sqrt{x+2})} \\
= \lim_{x \to a} \frac{a-x}{\sqrt{(a+2)(x+2)(x-a)(\sqrt{a+2} + \sqrt{x+2})}} \\
= \lim_{x \to a} \frac{-1}{\sqrt{(a+2)(a+2)(\sqrt{a+2} + \sqrt{x+2})}} \\
= \frac{-1}{2(a+2)(\sqrt{a+2})}. 
\]

20. This is \( f'(a) \) where \( f(x) = \sqrt{x} \) and \( a = 16 \).

22. This is \( f'(a) \) where \( f(x) = \tan x \) and \( a = \pi/4 \).

33. (a) \( S'(T) \) is the rate of change of solubility with respect to temperature. That is, it tells you how much more or less oxygen will dissolve in water that is a little bit cooler or warmer than the current temperature. (What effect will a change in temperature have on the solubility? Ask the derivative!) The units are in mg/L/°C. (Milligrams per liter per degree centigrade.

(b) \( S'(16) \) appears to be at \(-\frac{1}{4}\). It means that for each degree centigrade you raise the temperature above 16, you lose 1/4 of a milligram of oxygen per liter of water.