1. The Riemann sum represents the sum of the areas of the two rectangles above the $x$-axis minus the sum of the areas of the two rectangles below the $x$-axis. We have $R_4 = 0.5[(2 - 0.5^2) + (2 - 1^2) + (2 - 1.5^2) + 2 - 2^2)] = 0.25$.

3. With $n = 5$, we have $\Delta x = 1$. This Riemann sum represents the area under the last two rectangles minus the area under the first three rectangles. We have $(\sqrt{1.5} - 2) + (\sqrt{2.5} - 2) + (\sqrt{3.5} - 2) + (\sqrt{4.5} - 2) + (\sqrt{5.5} - 2) \approx -0.856759$.

5. Right endpoints give $\int_0^8 f(x)dx \approx 2(1 + 2 + (-2) + 1) = 4$. Left endpoints give $\int_0^8 f(x)dx \approx 2(2 + 1 + 2 + (-2)) = 6$. Using midpoints gives $\int_0^8 f(x)dx \approx 2(3 + 2 + 1 + (-1)) = 10$.

7. The left estimates are also lower estimates: $\int_0^{25} f(x)dx \approx 5(-42 - 37 - 25 - 6 + 15) = 475$. The right estimates are also upper estimates, giving $\int_0^{25} f(x)dx \approx 5(-37 - 25 - 6 + 15 + 36) = -85$.

9. $\int_0^{10} \sin \sqrt{x}dx \approx 2(\sin \sqrt{1} + \sin \sqrt{3} + \sin \sqrt{5} + \sin \sqrt{7} + \sin \sqrt{9}) \approx 6.4643$.

11. $\int_1^2 \sqrt{1 + x^2}dx \approx \sum_{i=1}^{10} 0.1(\sqrt{1 + (0.1i)^2}) \approx 1.8100$

17. We get $\int_0^\pi x \sin xdx$.

18. This is $\int_1^5 \frac{e^x}{1 + x}dx$.

19. This is $\int_0^1 (2x^2 - 5x)dx$.

20. This is $\int_1^4 \sqrt{x}dx$. 
21. 

\[ \int_{-1}^{5} (1 + 3x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left[ 1 + 3 \left( -1 + \frac{6}{n} i \right) \right] \frac{6}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} \left( -2 + \frac{18}{n} i \right) \frac{6}{n} \]

\[ = \lim_{n \to \infty} \left( -2n + \frac{18 n(n+1)}{2} \right) \frac{6}{n} \]

\[ = -12 + 54 \]

\[ = 42. \]

22. 

\[ \int_{1}^{5} (2 + 3x - x^2)dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + 3 \left( 1 + \frac{4}{n} i \right) - \left( 1 + \frac{4}{n} i \right)^2 \right) \frac{4}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} \left( 4 + \frac{4}{n} i - \frac{16}{n^2} i^2 \right) \frac{4}{n} \]

\[ = \lim_{n \to \infty} \left( 4n + \frac{4 n(n+1)}{2} - \frac{16 n(n+1)(2n+1)}{6} \right) \frac{4}{n} \]

\[ = 16 + 8 - \frac{64}{3} \]

\[ = \frac{8}{3}. \]

23. 

\[ \int_{0}^{2} (2 - x^2)dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left[ 2 - \left( \frac{2}{n} i \right)^2 \right] \frac{2}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \frac{4}{n} - \frac{8}{n^3} i^2 \right] \]

\[ = \lim_{n \to \infty} \left( 4 - \frac{8 n(n+1)(2n+1)}{6} \right) \]

\[ = 4 - \frac{16}{6} \]

\[ = \frac{4}{3}. \]

24. 

\[ \int_{0}^{5} (1 + 2x^3)dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left[ 1 + 2 \left( \frac{5}{n} i \right)^3 \right] \frac{5}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{5}{n} + 2 \cdot \frac{625}{n^4} i^3 \right) \]

\[ = \lim_{n \to \infty} \left( 5 + 625 \frac{n^2(n+1)^2}{2} \right) \]

\[ = 5 + 625 \]

\[ = \frac{635}{2}. \]
25. \[ \int_1^2 x^3 \, dx = \lim_{n \to \infty} \sum_{i=1}^n \left[ 1 + \frac{1}{n} \right]^{3/n} \frac{1}{n} \]
\[ = \lim_{n \to \infty} \sum_{i=1}^n \left[ 1 + \frac{3}{n} + \frac{3}{n^2} i^2 + \frac{1}{n^3} i^3 \right] \frac{1}{n} \]
\[ = \lim_{n \to \infty} \sum_{i=1}^n \left( \frac{1}{n} + \frac{3}{n^2} i + \frac{3}{n^3} i^2 + \frac{1}{n^4} i^3 \right) \]
\[ = \lim_{n \to \infty} \left( 1 + \frac{3}{n^2} \frac{n(n+1)}{2} + \frac{3}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{1}{n^4} \frac{n^2(n+1)^2}{4} \right) \]
\[ = 1 + \frac{3}{2} + 1 + \frac{1}{4} \]
\[ = \frac{15}{4}. \]

29. (a) The region is trapezoidal. We get \( \int_0^2 f(x) \, dx = 4. \)

(b) This region is made of a triangle, a rectangle, and a trapezoid. We get \( \int_0^5 f(x) \, dx = 10. \)

(c) This region is a triangle, but it is below the axis. Thus \( \int_5^7 f(x) \, dx = -3. \)

(d) Putting everything together gives \( \int_0^9 f(x) \, dx = 2. \)

30. (a) \( \int_0^2 g(x) \, dx = 4, \) the area of the triangular region.

(b) \( \int_2^6 g(x) \, dx = -2\pi, \) the negative of the area of the semicircular region.

(c) \( \int_0^7 g(x) \, dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi. \)

31. This is the area of a trapezoid with bases \( 1 + 2(1) = 3 \) and \( 1 + 2(3) = 7 \) and height 2 (the width of the interval). Thus, \( \int_1^3 (1 + 2x) \, dx = \frac{1}{2} (3 + 7)(2) = 10. \)

32. This the area of a semicircle with radius 2, so \( \int_{-2}^{2} \sqrt{4 - x^2} = \frac{1}{2} \pi (2)^2 = 2\pi. \)

33. This region is a quarter circle of radius 3 surmounting a rectangle of height 1 and base 3. Thus \( \int_{-3}^{0} (1 + \sqrt{9 - x^2}) \, dx = 3 + \frac{1}{4} \pi (3)^2 = 3 + \frac{9}{4} \pi. \)

34. This is two regions; one is a triangle of base 3 and height 3 above the x-axis, and the other is a triangle of base 1 and height 1 below the x-axis. We get \( \int_{-1}^{3} (2 - x) \, dx = \frac{1}{2} (3)(3) - \frac{1}{2} (1)(1) = 4. \)

35. This is three regions: two are triangles of base 1 and height 1 (each) below the x-axis, and one is a triangle of base 2 and height 1 above the x-axis. We get \( \int_{-2}^{2} (1 - |x|) \, dx = 2 \cdot \frac{1}{2} (1)(1) + \frac{1}{2} (2)(1) = 0. \)

36. This is made up of two regions, both of which are above the x-axis: one is a triangle of base \( \frac{5}{3} \) and height 5, and the other is a triangle of base \( \frac{4}{3} \) and height 4. We get \( \int_{0}^{3} |3x - 5| = \frac{1}{2} \frac{5}{3} (5) + \frac{1}{2} \frac{4}{3} (4) = \frac{11}{6}. \)
37. Since we are integrating backwards, we have \( \int_9^4 \sqrt{t} \, dt = -\frac{38}{3} \).

38. \( \int_1^1 x^2 \cos x \, dx = 0 \).

39. These all combine into \( \int_1^{12} f(x) \, dx \).

40. This is \( \int_2^{10} f(x) \, dx + \int_7^2 f(x) \, dx = \int_7^{10} f(x) \, dx \).

41. \( \int_2^5 f(x) \, dx = \int_2^8 f(x) \, dx + \int_8^5 f(x) \, dx = 1.7 + (-2.5) = -0.8 \).

42. \( \int_1^3 f(t) \, dt = \int_1^0 f(t) \, dt + \int_0^4 f(t) \, dt + \int_4^3 f(t) \, dt = -2 + (-6) + (-1) = -9 \).

48. Since \( \sqrt{x^3+1} \) is strictly increasing on \([0, 2]\), its minimum value is \( \sqrt{0^3+1} = 1 \) and its maximum value is \( \sqrt{2^3+1} = 3 \). Therefore, \( 0 = 0 \cdot 2 \leq \int_0^2 \sqrt{x^3+1} \, dx \leq 3 \cdot 2 = 6 \). The integral is between 0 and 6.

49. Notice that the terms in the summation have the form \( \left( \frac{1}{n} \right)^4 \frac{1}{n} \). Thus, it appears that the width of the interval is 1 and the starting point of the interval is 0. Also, since we are just taking the fourth powers of each term \( \frac{1}{n} \), the function seems to be \( x^4 \). The integral is therefore \( \int_0^1 x^4 \, dx \).