1. \[ \int_0^9 \frac{10}{(1 + x)^2} \, dx = -10 \left. \frac{1}{1 + x} \right|_0^9 = -1 + 10 = 9 \text{ foot-pounds.} \]

3. We must first determine the spring constant \( k \). We have \( k \cdot 4 = 10 \), so \( k = 2.5 \) pounds per inch. The work is \[ \int_0^6 2.5x \, dx = 1.25x^2 \bigg|_0^6 = 45 \text{ inch-pounds, or 3.75 foot-pounds.} \]

5. (a) We are told that \[ \int_0^{0.12} kxdx = 0.0072k = 2, \] so \( k = \frac{2500}{9} \text{ N/m.} \) Now \[ \int_{0.05}^{0.1} \frac{2500}{9} x \, dx = \frac{1250}{9}(0.01 - 0.0025) = \frac{25}{24} \text{ J.} \]

(b) Solve \( 30 = \frac{1}{36}x \) for \( x \). We get \( x = 1080 \text{ N-cm/kg, or 10.8 cm.} \)

7. If \( x \) feet of rope have been pulled up, then \( 50 - x \) feet, weighing \( \frac{1}{2}(50 - x) \) pounds, remain. The work done in pulling it up is therefore \[ \int_0^{50} \frac{1}{2}(50 - x) \, dx = 25x - \frac{1}{4}x^2 \bigg|_0^{50} = 625 \text{ ft-lbs.} \]

9. The length of cable remaining when the coal is raised \( x \) feet is \( 500 - x \) and weighs \( 1000 - 2x \) pounds. Thus, the total weight of the load is \( 1000 - 2x + 500 = 1800 - 2x \). The work done is \[ \int_{500}^{1500} (1500 - 2x) \, dx = 1800x - x^2 \bigg|_{500}^{1500} = 650,000 \text{ ft-lbs.} \]

11. The weight of a slab is \( (2)(1)(1000)(9.8) \Delta x = 19600 \Delta x \text{ N.} \) With \( x = 0 \) as the top of the aquarium, we get \( W = \int_0^{0.5} 19600x \, dx = 9800x^2 \bigg|_0^{0.5} = 2450 \text{ J.} \)

14. \( x \) feet below the top of the tank, the radius is \( \sqrt{b^2 - x^2} \). Therefore, the weight of a slab of water at this point is \( 62.5 \pi (\sqrt{b^2 - x^2})^2 \Delta x = 62.5 \pi (25 - x^2) \Delta x \). Pumping to the top requires \( W = \int_0^5 62.5 \pi (25 - x^2) x \, dx = 781.25 \pi x^2 - \frac{62.5 \pi}{4} x^4 \bigg|_0^5 = \frac{78125 \pi}{8} \approx 30680 \text{ ft-lbs.} \)

15. The work done over a small distance \( \Delta x \) is the force \( F = \pi r^2 P(V) \) applied over the \( \Delta x \), or \( \Delta W = \pi r^2 P(V) \Delta x \). Notice that \( \Delta V = \pi r^2 \Delta x \), so in fact \( \Delta W = P(V) \Delta V \). Summing over the range of volumes and taking limits give \( W = \int_{V_1}^{V_2} P(V) \, dV \).

17. (a) The work is \( W = \int_a^b \frac{Gm_1 m_2}{r^2} \, dr = \frac{Gm_1 m_2}{r} \left( \frac{1}{a} - \frac{1}{b} \right) \).

(b) Using part (a) and the given constants, we have \[ W = (6.67 \times 10^{-11})(1000)(5.98 \times 10^{24}) \left( \frac{1}{6.37 \times 10^6} - \frac{1}{7.37 \times 10^6} \right) \approx 8.5 \times 10^9 \text{ J.} \]

18. (a) To leave the Earth’s gravitational field, we must take the height to \( \infty \), so we get \( W = (6.67 \times 10^{-11})(1000)(5.98 \times 10^{24}) \left( \frac{1}{6.37 \times 10^6} \right) \approx 6.26 \times 10^{10} \text{ J.} \)

(b) The constant \( G \) does not change. The work is simultaneously \( \frac{1}{2} m v_0^2 \) and \( \frac{Gm}{R} \). Thus \( v_0 = \sqrt{\frac{2GM}{R}} \). Notice that this is independent of the mass of the rocket!
19. The pressure $x$ meters below the surface is $9800x$ ($\rho g x$). The cross-sectional area of a very small slice of the tank’s end is $2\sqrt{100 - x^2} \Delta x$ at this depth, so the force on the end of the tank is approximately
\[ \sum_{i=1}^{n} 9800x_i \cdot 2\sqrt{100 - x_i^2} \Delta x, \]
where I have adopted our usual conventions. The actual force is therefore
\[ \int_0^8 19600x\sqrt{100 - x^2} \, dx = -\frac{19600}{3} (100 - x^2)^{3/2} \bigg|_0^8 = 6.53 \times 10^6 \text{ J.} \]

20. The surface is 5 m lower than it was in 19, but otherwise the two are the same. Thus, we have
\[
\begin{align*}
\int_0^8 19600x\sqrt{100 - x^2} \, dx &= -\frac{19600}{3} (100 - x^2)^{3/2} \bigg|_0^8 = 6.53 \times 10^6 \text{ J.}
\end{align*}
\]

21. Drop perpendiculars from the ends of the upper edge to create a rectangle and two triangles; the triangles both have bases of length 4. Consider the triangle on the left. A line at height $x$ creates similar triangles, and the dimensions satisfy $x = \frac{w}{4}$, where $w$ is the base of the triangle at height $x$.
Thus $w = \frac{1}{2}x$, and the width of the tank at a depth of $x$ is therefore $\frac{1}{2}x + 12 + \frac{1}{2}x = 12 + x$.

Whew! Now we can determine the force on the end of the tank: $F = \lim_{n \to \infty} \sum_{i=1}^{n} 62.5x_i(12 + x_i) \Delta x = \int_0^8 62.5(12x + x^2) \, dx = 34667 \text{ lbs.}$

22. $M_y = 4(-1) + 8(2) = 12$, $M_x = 4(2) + 8(4) = 40$. The center of mass is at $\left(\frac{12}{12}, \frac{40}{12}\right) = (1, \frac{10}{3})$.

23. $M_y = 6(1) + 5(3) + 1(-3) + 4(6) = 42$; $M_x = 6(-2) + 5(4) + 1(-7) + 4(-1) = -3$. The center of mass is at $\left(\frac{42}{16}, -\frac{3}{16}\right)$.

24. Assume that $\rho = 1$. I estimate that the center of mass is at about $\left(1.3, 1.5\right)$. Compute: $M_x = \int_0^2 \frac{1}{2}x^2 \, dx = \frac{1}{10} x^5 \bigg|_0^2 = \frac{16}{5} = 3.2$. $M_y = \int_0^2 x(x^2) \, dx = \frac{1}{4} x^4 \bigg|_0^2 = 4$.

The area of the region is $\int_0^2 x^2 \, dx = \frac{1}{3} x^3 \bigg|_0^2 = \frac{8}{3}$. Therefore, the center of mass is at $\left(\frac{3.2}{8/3}, \frac{4}{8/3}\right) \approx (1.2, 1.5)$. Good for me!

25. In the interests of time, I will let you graph these yourselves. Here are my computations: $\overline{x} = \frac{\int_0^9 x\sqrt{x} \, dx}{\int_0^9 \sqrt{x} \, dx} = \frac{2}{3} (9)^{3/2} = \frac{27}{5}$. $\overline{y} = \frac{\int_0^9 \frac{1}{2}(\sqrt{x})^2 \, dx}{\int_0^9 \sqrt{x} \, dx} = \frac{1}{4} (9^2) = 9$. The center of mass is at $\left(5.4, 1.125\right)$. 


29. \[ A = \int_0^1 e^x \, dx = e^1 \bigg|_0^1 = e - 1. \] Thus \[ \bar{x} = \frac{\int_0^1 xe^x \, dx}{\int_0^1 e^x \, dx} = \frac{xe^x - e^1}{e - 1} = \frac{1}{e - 1}. \] \[ \bar{y} = \frac{\int_0^1 \frac{1}{2} e^{2x} \, dx}{\int_0^1 e^x \, dx} = \frac{1}{4} \frac{(e^2 - 1)}{e - 1} = \frac{1}{4} (e + 1). \]

30. \[ A = \int_1^2 \frac{1}{x} \, dx = \ln x \bigg|_1^2 = \ln 2. \] Thus \[ \bar{x} = \frac{\int_1^2 dx}{\ln 2} = \frac{1}{\ln 2}. \] \[ \bar{y} = \frac{\int_1^2 \frac{1}{2} x^2 \, dx}{\ln 2} = -\frac{1}{2} \ln 2 \cdot \frac{1}{x} \bigg|_1^2 = -\frac{1}{4} \ln 2. \]

31. Because of the symmetry about the \( x \)-axis, the moment about the \( y \)-axis is 0 and \( \bar{x} = 0 \). The moment about the \( x \)-axis is \[ 2 \int_0^1 \frac{1}{2} (2 - 2x)^2 \, dx = -\frac{1}{6} (2 - 2x)^3 \bigg|_0^1 = \frac{4}{3}. \] The mass is \( m = \frac{1}{2} (2)(2) = 2 \), so \( \bar{y} = \frac{2}{3} \).

32. Because of the symmetry, the moments about the \( x \)- and \( y \)-axes will be equal and \( \bar{y} = \bar{x} \). The moment about the \( x \)-axis is \[ 2 \int_0^r x \sqrt{r^2 - x^2} \, dx = 2 \int_0^r x (r^2 - x^2)^{1/2} \, dx = -\frac{2}{3} (r^2 - x^2)^{3/2} \bigg|_0^r = \frac{2}{3} r^3. \] \[ m = 2 \frac{1}{4} \pi r^2 = \frac{1}{2} \pi r^2, \] so \( \bar{x} = \frac{4}{3\pi} r. \)