The region in the $xy$-plane is below the graph of $y = \sqrt{x}$, above the $x$-axis, and to the left of $x = -1$.

11. The region of integration is graphed below. We have

$$\int_0^1 \int_0^2 \int_0^{e^{-z^2}} x e^y dxdydz = \int_0^1 \int_0^2 \int_0^{e^{-z^2}} \frac{1}{2} z (e^{-z^2} - 1) dz = \frac{1}{4} e^{-z^2} + \frac{1}{4} z^2 \bigg|_0^1 = \frac{1}{4} (1 + e^{-1} - 1) = f14e.$$

12. The region is the tetrahedron in the first octant with vertices $(0, 0, 0), (2, 0, 0), (0, 2, 0)$, and $(0, 0, 4)$. In the $xy$-plane, the region is bounded by the line $y = 2 - x$. (Set $z = 0$ in the plane bounding the region above.) We have

$$\int_0^1 \int_0^2 \int_0^{2-x} \frac{1}{2} (2 - x)^3 dx = \int_0^2 (2 - x)^2 - \frac{2}{3} (2 - x)^3 dx = \int_0^2 2(2 - x)^2 - (4x - 4x^2 + x^3) - \frac{2}{3} (2 - x)^3 dx = \frac{2}{3} (2 - x)^3 - 2x^2 + \frac{4}{3} x^3 - \frac{1}{4} x^4 + \frac{1}{6} (2 - x)^4 \bigg|_0^2 = \left(-8 + \frac{32}{3} - 4\right) - \left(-\frac{16}{3} + \frac{8}{3}\right) = \frac{4}{3}.$$

13. You don’t really need to see me integrate polynomials, so I will just set the rest of these up. The cylinder is an upside-down parabolic cylinder. It meets the plane $z = 0$ at $y = \pm 1$, so that gives the range for $y$. We get

$$\int_1^0 \int_{-1}^1 \int_{-y^2}^{y^2} x^2 e^y dxdydz = \frac{8}{3e}.$$

19. The boundary in the $xy$-plane is $y = 4 - 2x$, so $x$ ranges from 0 to 2 and the volume is

$$\int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dzdxdy = \frac{16}{3}.$$

21. This is a natural candidate for polar coordinates. We have

$$\int_0^{2\pi} \int_0^3 (\int_0^2 r^2 e^{-r^2} dr) r dr d\theta = \int_0^{2\pi} \int_0^3 (5r - 5r^2 \sin \theta - r dr d\theta = \int_0^{2\pi} (2) \frac{5}{3} (27) \sin \theta d\theta = 36\pi.$$

12. The region in the $xy$-plane is below the graph of $y = \sqrt{x}$, above the $x$-axis, and to the left of $x = -1$. The region of integration is graphed below. We have

$$\int_0^1 \int_0^2 \int_0^{e^{-z^2}} x e^y dxdydz = \int_0^1 \int_0^2 \int_0^{e^{-z^2}} \frac{1}{2} z (e^{-z^2} - 1) dz = \frac{1}{4} e^{-z^2} + \frac{1}{4} z^2 \bigg|_0^1 = \frac{1}{4} (1 + e^{-1} - 1) = f14e.$$

12. The region in the $xy$-plane is below the graph of $y = \sqrt{x}$, above the $x$-axis, and to the left of $x = -1$. The region of integration is graphed below. We have

$$\int_0^1 \int_0^2 \int_0^{e^{-z^2}} x e^y dxdydz = \int_0^1 \int_0^2 \int_0^{e^{-z^2}} \frac{1}{2} z (e^{-z^2} - 1) dz = \frac{1}{4} e^{-z^2} + \frac{1}{4} z^2 \bigg|_0^1 = \frac{1}{4} (1 + e^{-1} - 1) = f14e.$$
23. \( \int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz \, dy \, dx \). This can be evaluated with a trigonometric substitution, but I will follow the book's advice and ask MAPLE.

(b) MAPLE says... \( \frac{1}{4} \pi - \frac{1}{3} \).

27. While \( x \) ranges from 0 to 1, \( z \) ranges from 0 to the plane \( 1 - x \), which slants down over the \( xy \)-plane to meet it in the line \( x = 1 \). \( y \) ranges from 0 to \( 2 - 2z \); the latter gives a plane slanting down and meeting the \( xy \)-plane in the line \( y = 2 \). The result is a square-based pyramid with vertices \((0, 0, 0), (1, 0, 0), (0, 2, 0), (1, 2, 0), \) and \((0, 0, 2)\). See the back of the book for a graph.