Find the power series expansion of \( f(x) = \arctan(x) \) about \( x = 0 \). Recall that \( f'(x) = \frac{1}{1 + x^2} \).

**Solution:** We have \( f'(x) = \frac{1}{1 + x^2} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n} \). Thus

\[
f(x) = \int \sum_{n=0}^{\infty} (-1)^n x^{2n} \, dx = \sum_{n=0}^{\infty} \int (-1)^n x^{2n} \, dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} + C.
\]

Since \( f(0) = 0 \), we must have \( C = 0 \), so \( \arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \).