Use power series methods to solve \((x^2 - 1)y'' - 2xy' + 2y = 0\) about \(x = 0\).

**Solution:** Let \(y = \sum_{n=0}^{\infty} a_n x^n\). Then \(y' = \sum_{n=1}^{\infty} a_n n x^{n-1}\) and \(y'' = \sum_{n=2}^{\infty} a_n (n-1) x^{n-2}\). We get

\[(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1) \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} - 2x \sum_{n=1}^{\infty} a_n n x^{n-1} + 2 \sum_{n=0}^{\infty} a_n x^n\]

\[= \sum_{n=2}^{\infty} a_n n(n-1) x^n - \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} - \sum_{n=1}^{\infty} 2a_n n x^n + \sum_{n=0}^{\infty} 2a_n x^n\]

\[= \sum_{n=2}^{\infty} a_n n(n-1) x^n - \sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1) x^n - \sum_{n=1}^{\infty} 2a_n n x^n + \sum_{n=0}^{\infty} 2a_n x^n\]

\[= (-a_2 \cdot 2 \cdot 1 + 2a_0) + (-a_3 \cdot 3 \cdot 2 - 2a_1 \cdot 1 + 2a_1)x\]

\[+ \sum_{n=2}^{\infty} (a_n n(n-1) - a_{n+2}(n+2)(n+1) - 2a_n n + 2a_n) x^n\]

\[= 0.\]

Thus \(a_2 = a_0, a_3 = 0,\) and \(a_{n+2} = \frac{a_n [n(n-1) - 2n - 2]}{(n+2)(n+1)} = \frac{a_n (n-1)(n-2)}{(n+1)(n+2)}.\) We get \(a_0 = a_0, a_1 = a_1, a_2 = a_0, a_3 = 0, a_4 = \frac{a_2 (2-1)(2-2)}{(2+1)(2+2)} = 0, a_5 = 0, \ldots.\) Therefore, \(y(t) = a_0 + a_1 x + a_0 x^2,\) or just \(a_0(1 + x^2) + a_1 x.\)