1. Find the Laplace transform of each function. Since I gave you a table, I will need to see your work!

(a) \( f(t) = t \).

Solution: \( \mathcal{L}\{t\} = \int_0^\infty te^{-st} \, dt = -\frac{1}{s} \left( t e^{-st} + \frac{1}{s} e^{-st} \right) \bigg|_0^\infty = -\frac{1}{s} \left( 0 - \frac{1}{s} \right) = \frac{1}{s^2} \).

(b) \( f(t) = e^{at} \).

Solution: \( \mathcal{L}\{e^{at}\} = \int_0^\infty e^{at} e^{-st} \, dt = \frac{1}{a-s} e^{(a-s)t} \bigg|_0^\infty = \frac{1}{a-s} (0 - 1) = \frac{1}{s-a} \).

2. Solve \( y'' + 4y' + 4y = 0, \ y(0) = 0, y'(0) = 1 \) using Laplace transform methods.

Solution: Apply the Laplace transform to both sides to obtain

\[
\mathcal{L}\{y'' + 4y' + 4y\} = 0 \\
[s^2Y(s) - sy(0) - y'(0)] + 4[Y(s) - y(0)] + 4Y(s) = 0 \\
(s^2 + 4s + 4)Y(s) - 1 = 0 \\
Y(s) = \frac{1}{(s+2)^2}.
\]

Thus, using Number 11 from the table, we see that \( f(t) = te^{-2t} \).