Problem 1:

Consider the region \( R \) bounded by the curves

\[
 y = -x^2 + 10, \quad y = -x + 4.
\]

(a) Sketch the curves given above, label the points of intersection, and shade the region.

(b) Set up but DO NOT EVALUATE an integral which calculates the volume of the solid obtained by revolving the region about the \( x \)-axis.

(c) Set up but DO NOT EVALUATE an integral which calculates the volume of the solid obtained by revolving the region about the line \( x = 4 \).

(d) Set up but DO NOT EVALUATE an integral which calculates the volume of the solid with base \( R \) and cross sections perpendicular to the to the \( x \)-axis are semi-circles.
Problem 2: Suppose the region bounded by the curve \( y = x \sin(\frac{x}{4}) \), \( 0 \leq x \leq 4\pi \), and \( y = 0 \) is rotated about the \( y \)-axis. Find the volume of the resulting solid. Evaluate all trig expressions in your final answer. A graph of \( y = x \sin(\frac{x}{4}) \) is given below.
Problem 3: Breathing is cyclic and a full respiratory cycle from the beginning of the inhalation to the end of the exhalation takes about 5 seconds. The maximum rate of of air flow into the lungs is about 0.5L/sec. This explains, in part, why the function \( f(t) = \frac{1}{2} \sin \left( \frac{2\pi t}{5} \right) \) has often been used to model the rate of air flow in the lungs.

a) Use the model above to find the volume of inhaled air in the lungs at time \( t \).

b) Use part a) to find the average volume of inhaled air in the lungs in one respiratory cycle.