Math 142

Professor Johnson

Problem 1: (a) Find the 4th degree Taylor polynomial for \( f(x) = \sqrt{x} \) centered at \( a = 9 \). You needn’t simplify your answer by multiplying out the fractions.

(b) Use the 4th degree Taylor polynomial to approximate \( \sqrt{9.3} \) and describe the error. You may assume that the Taylor Series for \( f(x) \) centered at \( a = 9 \) is equal to \( \sqrt{x} \) for all x-values in an interval that contains \( x = 9.3 \).
Problem 2: (a) Find the power series representation of \( g(x) = \ln(1 + x) \) centered at \( a = 0 \). Show all work.

(b) Determine the interval of convergence of the power series representation of \( g(x) \) found above.

(c) Find the sum of the series. Hint: Use parts (a) and (b).

\[
\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(0.8)^n}{n} = \\
\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} =
\]
Problem 3: (a) Find the Maclaurin series for \( g(x) = x^2 e^{(x^{10})} \). By following the steps below.

(i) First find the Maclaurin series for \( e^x \) and determine its interval of convergence.

(ii) Plug \( x^{10} \) into the series representation for \( e^x \) found above to find a power series representation for \( e^{x^{10}} \).

(iii) Multiply and distribute \( x^2 \) times the series representation for \( e^{x^{10}} \) found above.

(b) Use the Maclaurin series for \( g(x) = x^2 e^{(x^{10})} \) to evaluate the integral \( \int_{-0.3}^{0} x^2 e^{(x^{10})} \, dx \). Your answer will be a series.

(c) Give an example of another integral that cannot be evaluated using any of the previously studied techniques, but CAN be evaluated using the power series technique above.